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Coding against correlated synchronization errors

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Based on joint work with

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Technion

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UIUC/Purdue

Synchronization errors

Erasures

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↓

1 0 ? 0 1 ? ? 0

Deletions

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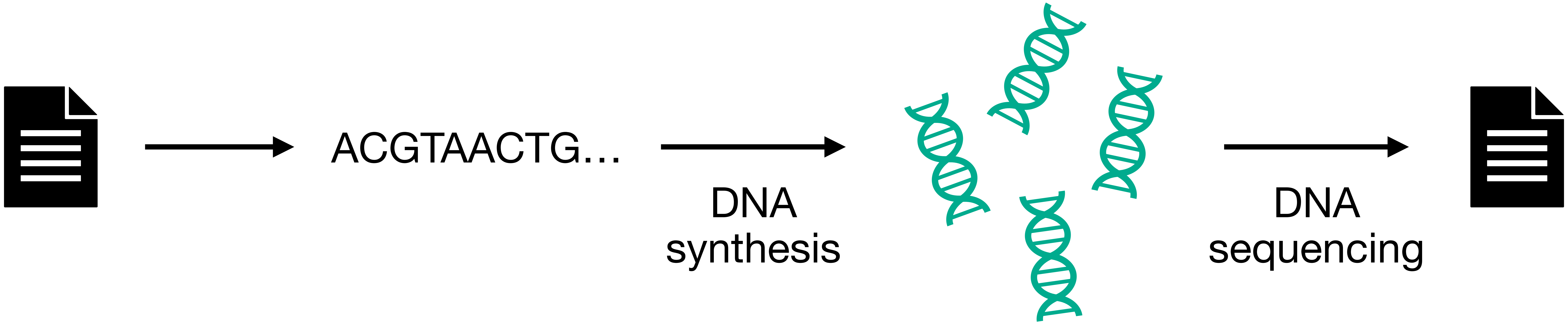
Why deletions and insertions?

- Simple types of synchronization errors.
- Most existing techniques fail.
- DNA-based data storage systems.



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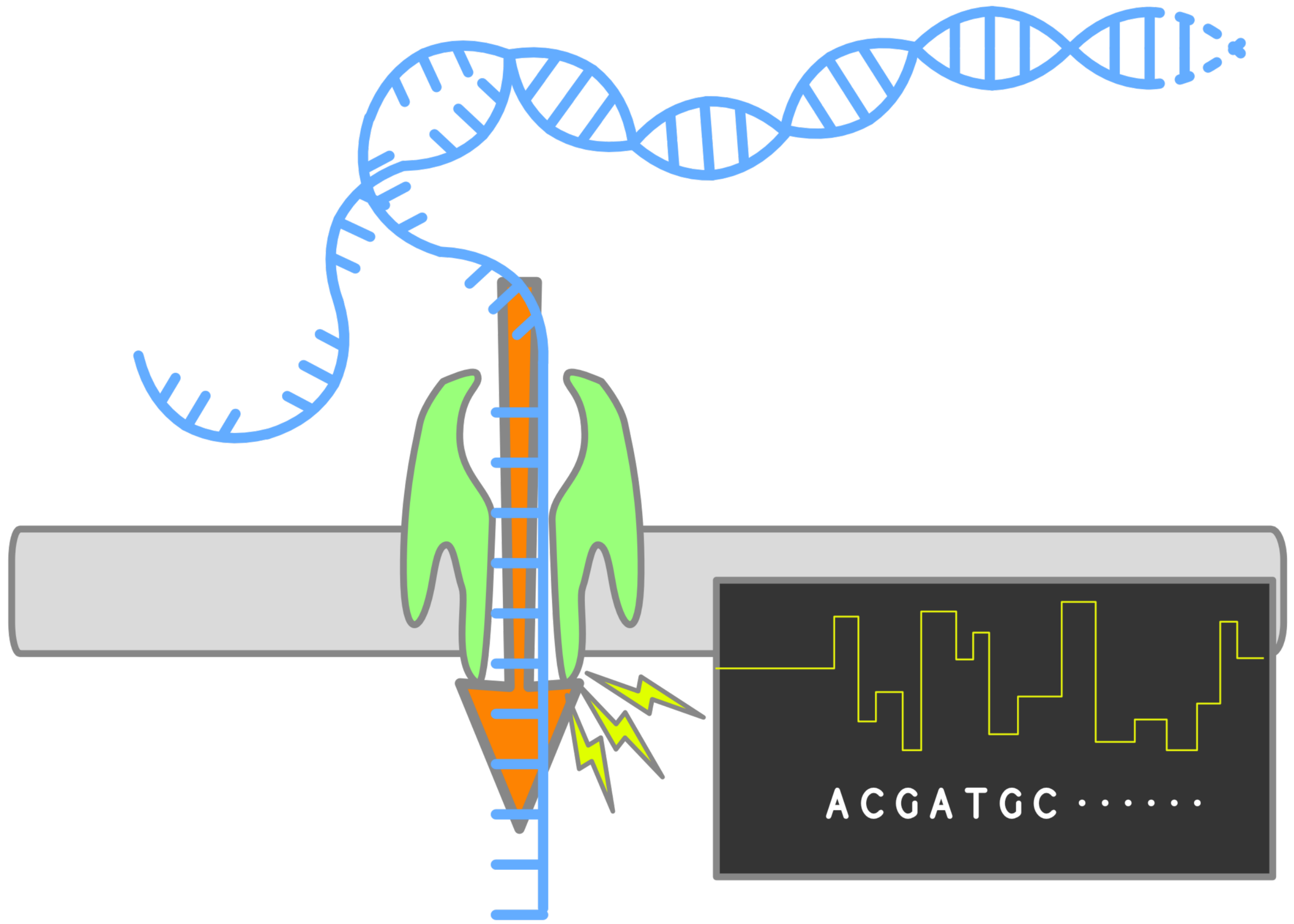
- **But the world is not iid! Error distribution can be affected by the environment and structure of input.**
  This was pointed out already in early works on DNA storage.

Concrete examples

In DNA-based data storage with nanopore-based sequencing, deletions happen more often in or around long runs.

[Yazdi, Gabrys, Milenkovic 2017], [Weindel, Gimpel, Grass, Heckel 2023]



Credit: DataBase Center for Life Science (DBCLS), https://commons.wikimedia.org/wiki/File:202001_nanopore_sequencing.svg

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Runlength-dependent deletion probabilities

Channel parameter: $d : \mathbb{N} \to [0,1]$

Behavior: bit in run of length ℓ is independently deleted with probability $d(\ell)$

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Dobrushin 1967: true for i.i.d. synchronization errors.

Mao-Diggavi-Kannan 2018: true for "finite-window" contextual synchronization errors.

Given $x \in \{0,1\}^n$, channel outputs $Y_1 \| Y_2 \| \cdots \| Y_n$ with $Y_i \in \{0,1\}^*$ dependent on $x_i, x_{i-1}, \ldots, x_{i-w}$ for some constant w.

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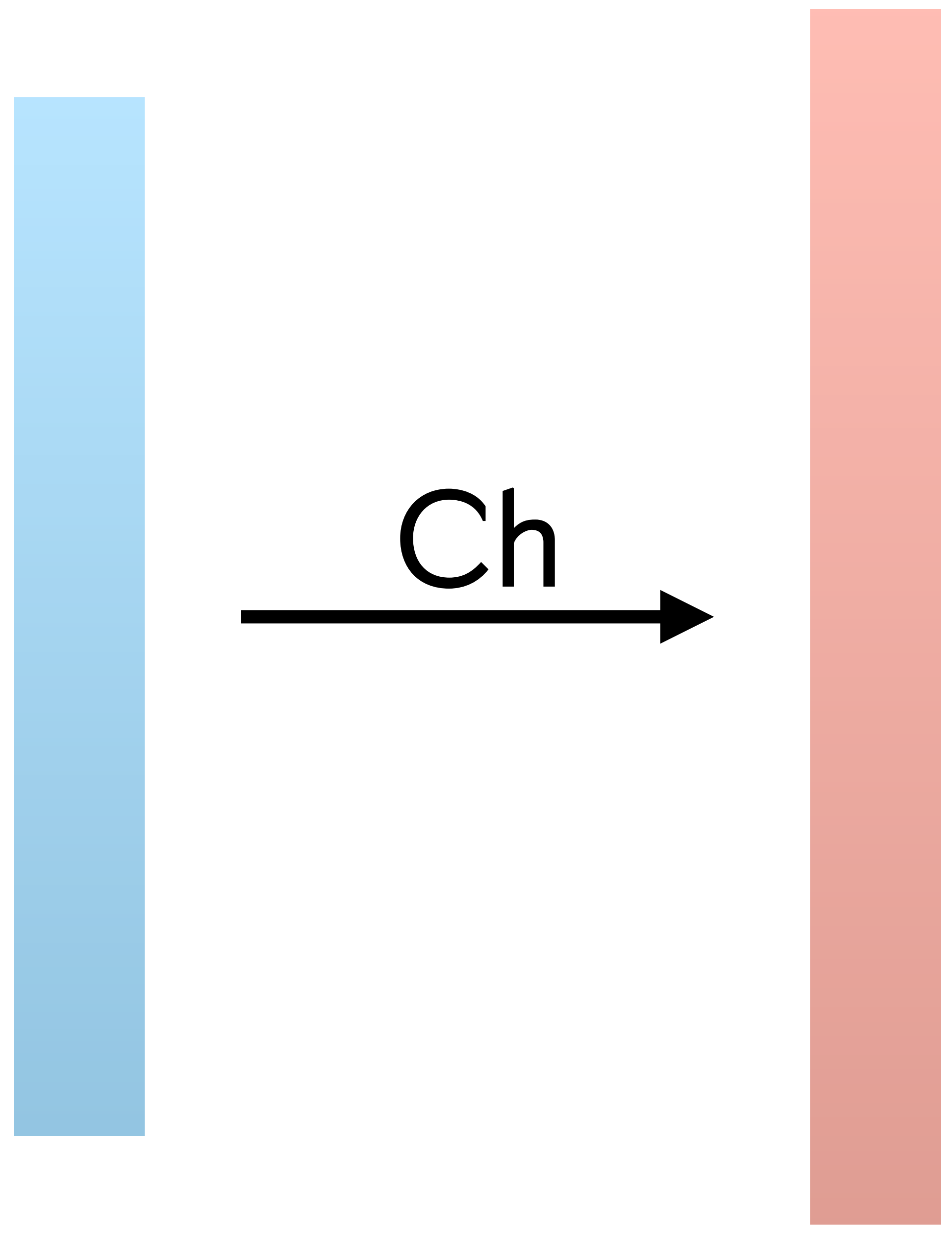

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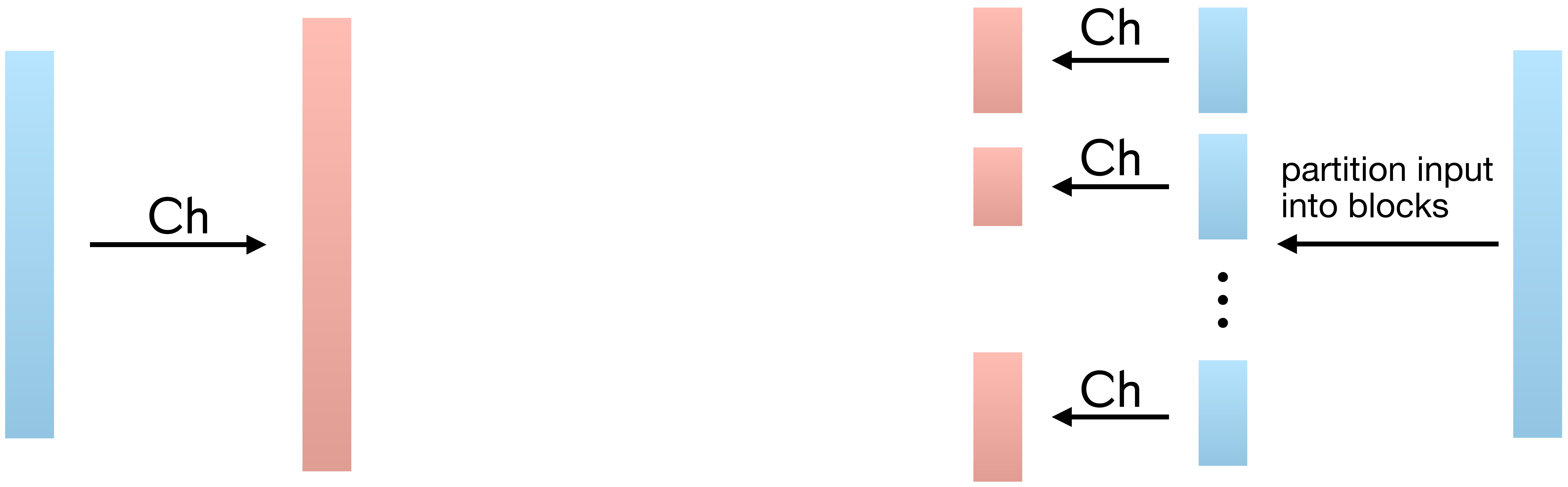

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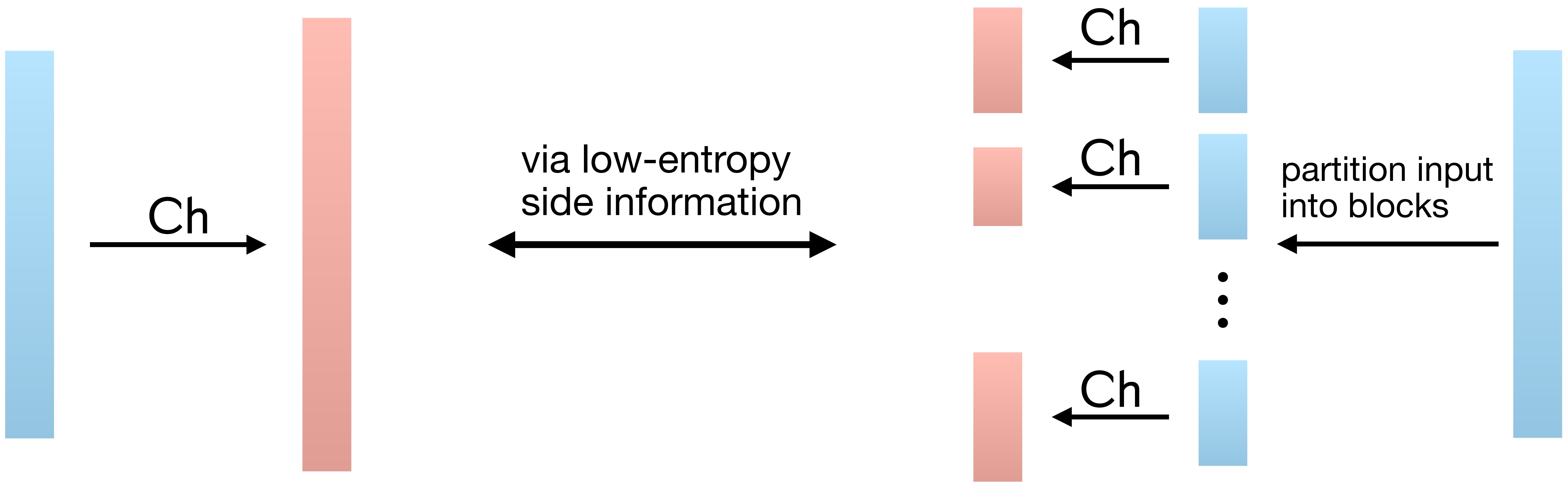

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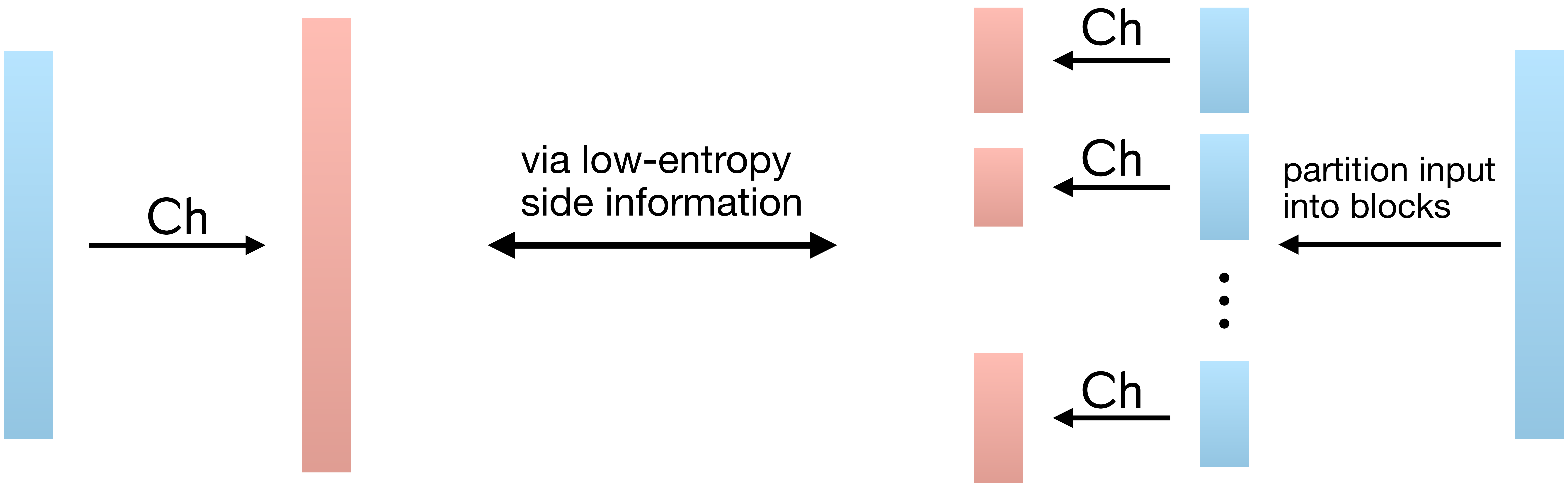

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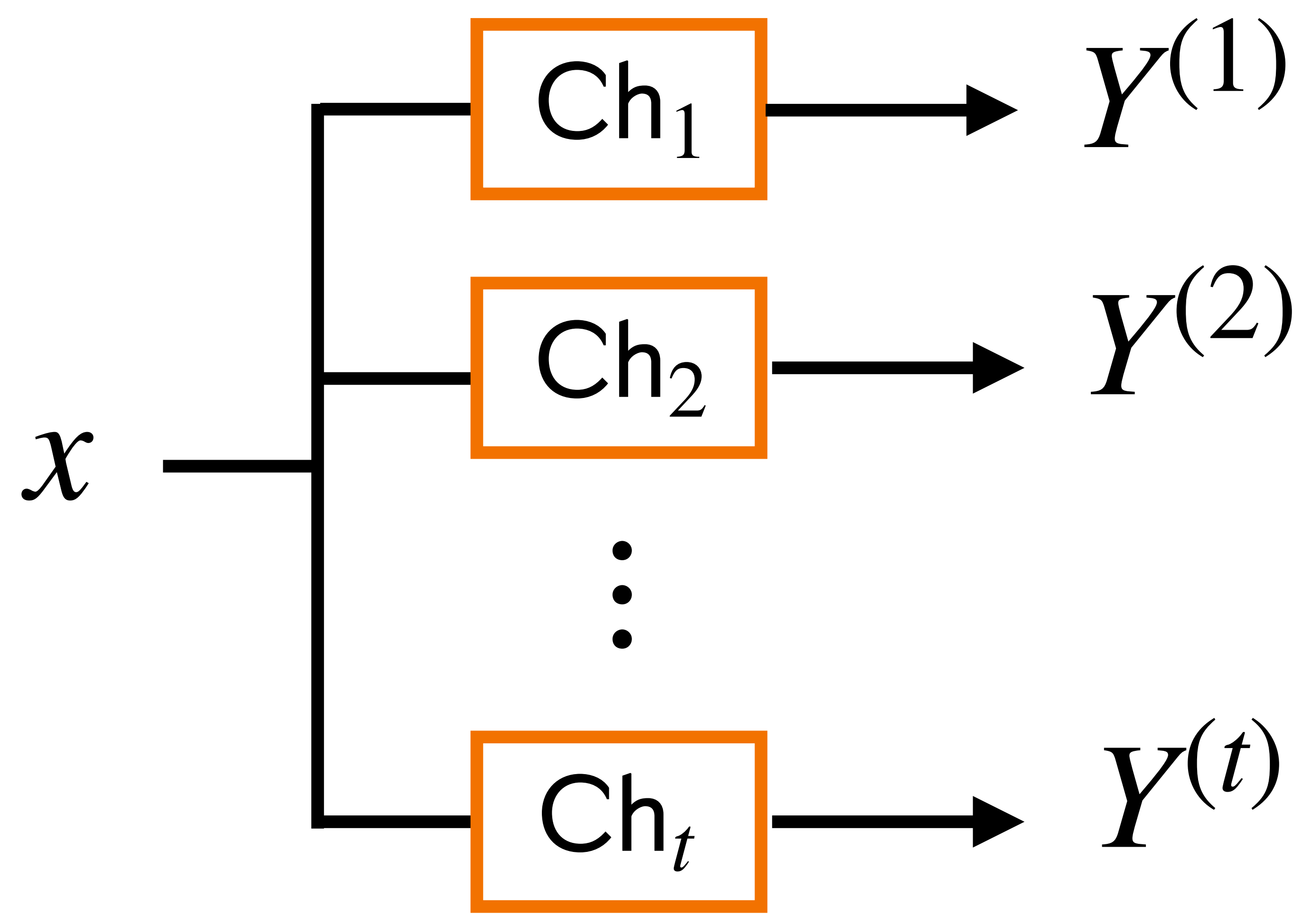

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Fix a channel with ***runlength-dependent*** deletions. Then, there exists a family of codes $(C_n)_{n\in\mathbb{N}}$ that:

- Achieves capacity on this channel;
- Is encodable in linear time and decodable in quasi-linear time in n;
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With more effort, can extend this result to the multi-trace setting

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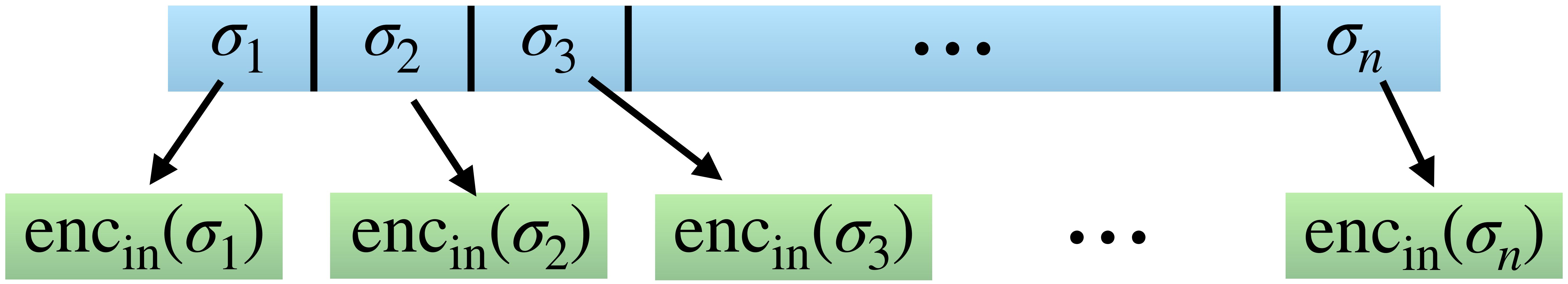

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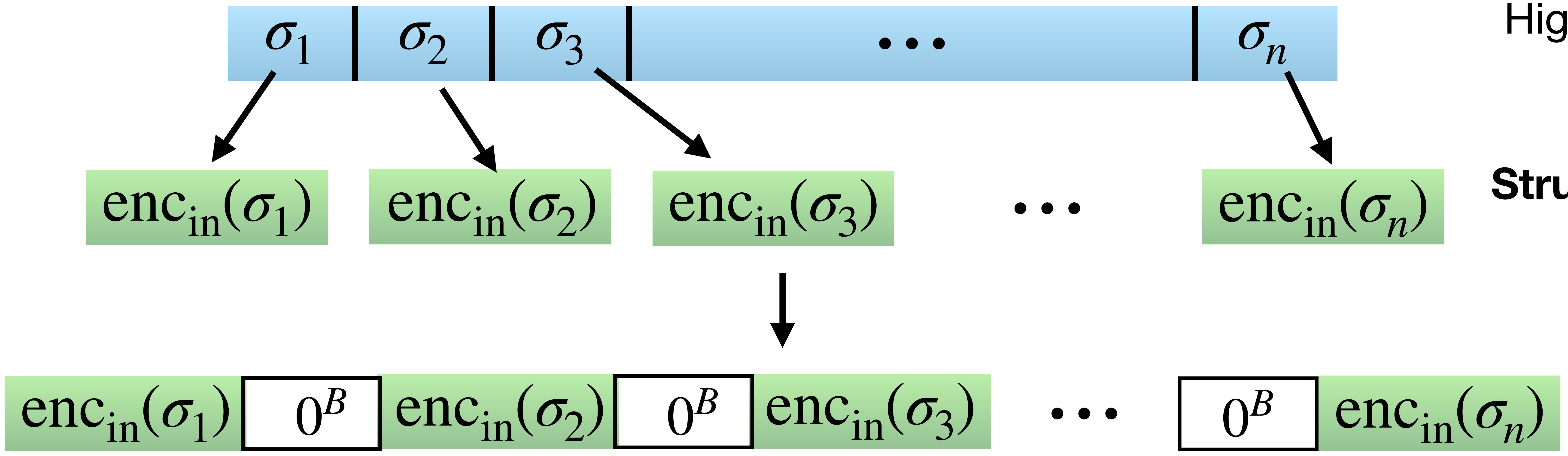

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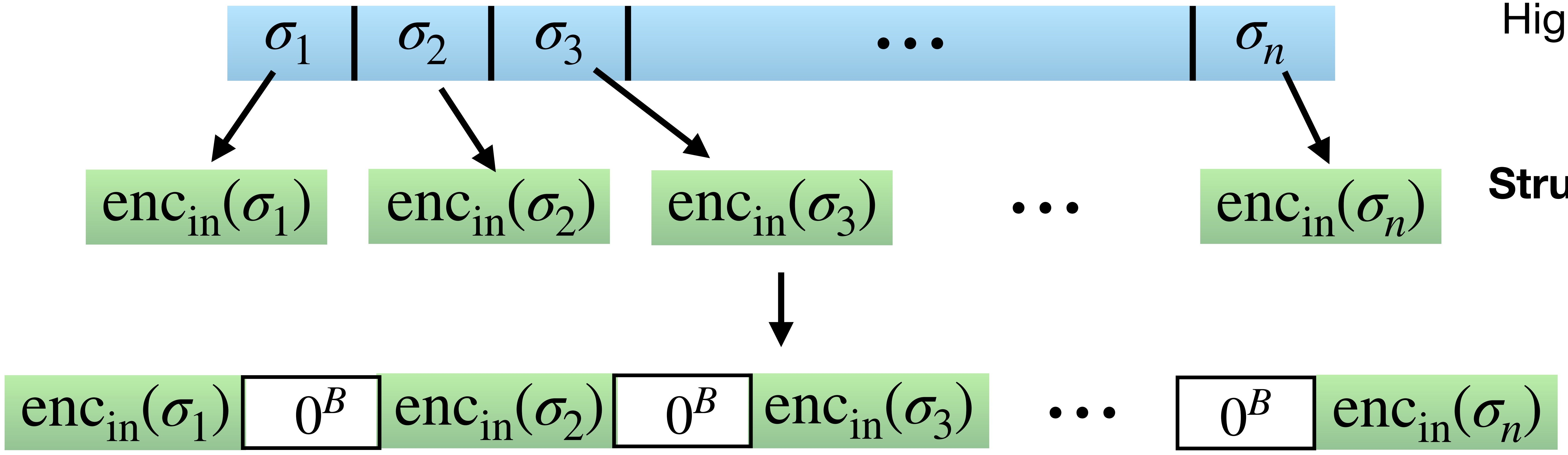

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concatenation + marker-based techniques are robust to changes in error model!

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with Yuan-Pon Chen, Olgica Milenkovic, Jin Sima

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We still haven't explored the effect of long runs on surrounding bits in DNA storage.

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t-deletion-correcting codes work here, but can we do better by exploiting contextuality?

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Can actually achieve this redundancy with efficient encoding/decoding!

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Better bounds on redundancy?

Lower thresholds?